Inference of particle sensor measurements: Beyond analytic models

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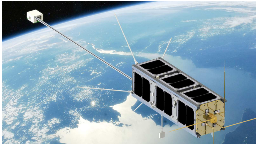
Outline

Thanks to:

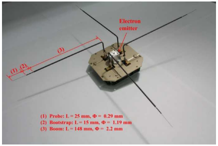
Joshua Gutrhie
Guangdong Liu
Sigvald Marholm
Akinola Olowookere
Pedro Alberto Resendiz Lira

The problem

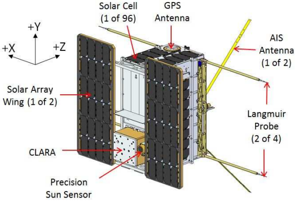
- Many satellites carry Langmuir probes.
- Density and temperature are not measured directly.
- They are inferred from measured currents vs. voltages.



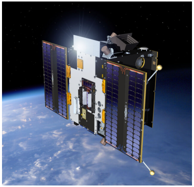
Ex-Alta 1



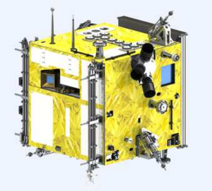
Fixed-bias Langmuir probes



Norsat 1 & 2



Proba-2



Chinese Seismoelectromagnetic Satellite

NASA Data processing levels[1]

Level	Description
L0	Reconstructed, unprocessed instrument and payload data at full resolution, with any and all communications artifacts (e.g., synchronization frames, communications headers, duplicate data) removed. (In most cases, NASA's EOS Data and Operations System [EDOS] provides these data to the Distributed Active Archive Centers [DAACs] as production data sets for processing by the Science Data Processing Segment [SDPS] or by one of the Science Investigator-led Processing System [SIPS] to produce higher-level products.)
L1A	Reconstructed, unprocessed instrument data at full resolution, time-referenced, and annotated with ancillary information, including radiometric and geometric calibration coefficients and georeferencing parameters (e.g., platform ephemeris) computed and appended but not applied to Level 0 data.
L1B	Level 1A data that have been processed to sensor units (not all instruments have Level 1B source data).
L2	Derived geophysical variables at the same resolution and location as Level 1 source data.
L3	Variables mapped on uniform space-time grid scales, usually with some completeness and consistency.
L4	Model output or results from analyses of lower-level data (e.g., variables derived from multiple measurements).

[1] earthdata.nasa.gov/collaborate/open-data-services-and-software/ ...

Inference models

In practice probe data analyses are based on theories leading to analytic inference algorithms as, for example, in.

- Orbital motion limited (OML) theory
- Radial motion theory.

These can be applied to

- small spherical probes,
- thin and long cylindrical probes,
- planar probes.

Custom models have also been developed for planar probes, accounting for

- fringe effects
- Effective collecting cross sections.

OML current collected by a sphere: $V < 0$

$$
\begin{aligned}
I_{net} &= \pi a^2 en\sqrt{\frac{2kT_i}{\pi m_i}}\left[e^{-x_{id}^2} + \left(1 + 2x_{id}^2 - x_{im}\right)\frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(x_{id})}{x_{id}}\right] \\
&- \pi a^2 en\sqrt{\frac{2kT_e}{\pi m_e}}\left\{\frac{x_{ed} + x_{em}}{2x_{ed}}e^{-(x_{ed}-x_{em})^2} + \frac{x_{ed} - x_{em}}{2x_{ed}}e^{-(x_{ed}+x_{em})^2}\right. \\
&+ \left.\left[\frac{1}{2} + x_{ed}^2 + x_{em}\right]\frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(x_{ed} - x_{em}) + \mathrm{erf}(x_{ed} + x_{em})}{x_{ed}}\right\}
\end{aligned}
$$

For a given species

$$x_{\alpha d} = \frac{v_{\alpha d}}{\sqrt{2kT_\alpha / m_\alpha}}$$

$$x_{\alpha m} = q_\alpha V / kT_\alpha$$

$$I_{net} \simeq nea^2\sqrt{\frac{8\pi kT_i}{m_i}}\left(1 - \frac{eV}{kT_i}\right) \text{ if } v_{id} = 0$$

$$I_{net} \simeq \pi nea^2 v_{id}\left(1 - \frac{eV}{mv_{id}^2/2}\right) \text{ if } v_{id} \gg v_{i\,th}$$

OML current collected by a sphere: $V > 0$

$$
\begin{aligned}
I_{net} &= \pi a^2 en\sqrt{\frac{2kT_i}{\pi m_i}}\left\{\frac{x_{id}+x_{im}}{2x_{id}}e^{-(x_{id}-x_{im})^2}+\frac{x_{id}-x_{im}}{2x_{id}}e^{-(x_{id}+x_{im})^2}\right.\\
&+ \left.\left[\frac{1}{2}+x_{id}^2-x_{im}\right]\frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(x_{id}-x_{im})+\mathrm{erf}(x_{id}+x_{im})}{x_{id}}\right\}\\
&- \pi a^2 en\sqrt{\frac{2kT_e}{\pi m_e}}\left[e^{-x_{ed}^2}+\left(1+2x_{ed}^2+2x_{em}\right)\frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(x_{ed})}{x_{ed}}\right]
\end{aligned}
$$

If $v_{ed} \simeq 0$, and $eV > kT_e$,

$$I \simeq nea^2\sqrt{\frac{8\pi kT_e}{m_i}}\left(1+\frac{eV}{kT_e}\right)$$

OML current collected by a thin cylinder: $eV > kT$

- For an thin, infinitely long probe,

$$
\begin{aligned}
I_{attracted} &\simeq -n_e e A \frac{2}{\sqrt{\pi}} \sqrt{\frac{kT_e}{2\pi m_e}} \sqrt{1 + \frac{e(V_f + V_b)}{kT_e}}, \\
\Rightarrow \quad n_e &= \sqrt{\frac{\pi m_e (\Delta I^2 / \Delta V_b)}{2e^3 A^2}}.
\end{aligned}
$$

(Jacobsen, Measurement Science and Technology, 2010)

- For a finite length cylinder however,

$$
I_{attracted} \simeq -n_e e A \sqrt{\frac{kT_e}{2\pi m_e}} \left(1 + \frac{e(V_f + V_b)}{kT_e}\right)^{\beta},
$$

with

$$
0.5 \lesssim \beta \lesssim 1.
$$

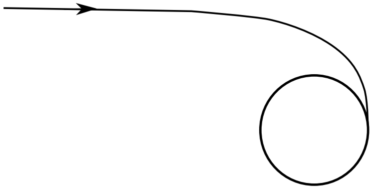
Common assumptions made in OML

- $r \ll \lambda_D$
- For a cylindrical probe, $l \gg \lambda_D$
- Stationary electrons
- Either $v_i \gg v_{i\,th}$ or $v_i \ll v_{i\,th}$
- $\vec{B} = 0$
- No collisions
- Stationary and uniform plasma background
- No nearby objects.
- No photoelectron or secondary electron emission
- Maxwellian background distribution

Problem: Some of these assumptions are not satisfied.

Radial motion limited[2]

This model is partly analytic and partly numerical.



- Similar to OML: It makes use of
  - conservation of energy $mv^2/2 + qV$, and
  - conservation of angular momentum $m\vec{r} \times \vec{v}$.
- Particles are affected by Debye shielding however.
  $\Rightarrow$ The maximum impact parameter doesn't necessarily correspond to grazing particles collected at the back of the probe.
- Poisson's equation needs to be solved numerically.

[2] J.E. Allen, Physica Scripta **45** (1992)

Planar probes

$$
\begin{aligned}
J &= en\sqrt{\frac{2kT}{\pi m_i}}\left[\frac{1}{2}e^{-x_{id}^2} + \frac{\sqrt{\pi}}{2}x_{id}\left(1+\mathrm{erf}(x_{id})\right)\right] \\
&- en\sqrt{\frac{2kT}{\pi m_e}}\left[\frac{1}{2}e^{-(x_{ed}-x_{em})^2} + \frac{\sqrt{\pi}}{2}x_{ed}\left(1+\mathrm{erf}(x_{ed}-x_{em})\right)\right]
\end{aligned}
$$

if $V < 0$, and

$$
\begin{aligned}
J &= en\sqrt{\frac{2kT}{\pi m_i}}\left[\frac{1}{2}e^{-(x_{id}-x_{im})^2} + \frac{\sqrt{\pi}}{2}x_{id}\left(1+\mathrm{erf}(x_{id}-x_{im})\right)\right] \\
&- en\sqrt{\frac{2kT}{\pi m_e}}\left[\frac{1}{2}e^{-x_{ed}^2} + \frac{\sqrt{\pi}}{2}x_{ed}\left(1+\mathrm{erf}(x_{ed})\right)\right]
\end{aligned}
$$

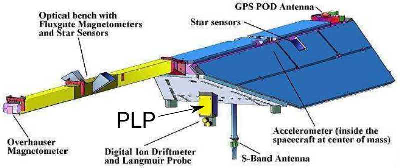
if $V > 0$.

Problem: Edge effects increase/decrease the effective collecting cross section for attracted/repelled species.

Corrections

- Fringe electric fields
  - enhance the effective cross section for attracted species, and
  - decrease it for repelled species.
- Analytic and empirical corrections have been reported:
  - Johnson, Holmes, Rev. Sci. Instrum. Vol. 61, 2628 (1990); doi: 10.1063/1.1141849
  - Sheridan, J. Phys. D: Applied Physics (2010)

Custom model: CHAMP PLP



$$I_P = -e n_e v_{\text{orbit}} A_P \left[\frac{\frac{A_P}{A_{\text{Se}}} + \frac{A_{\text{Si}}}{A_{\text{Se}}}}{\frac{A_P}{A_{\text{Se}}} + e^{-\frac{eV}{kT_e}}} - 1 \right]$$

(Rother, et al., Radio Sci. Vol. 45, 2010)

- A_P = probe surface area
- A_{Se} = satellite effective electron collection area
- A_{Si} = satellite effective ion collection area
- V = plate potential with respect to the satellite

Problems:

- Edge effects increase/decrease the effective collecting cross section for attracted/repelled species.
- Nearby objects and magnetic field are not accounted for.

What other options are there?

To be practical, these options must satisfy the following criteria:

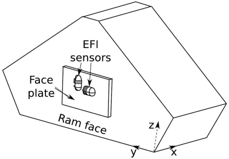
- Higher accuracy than analytic expression. More physical processes and general conditions under which measurements are made, must be accounted for.
- Speed: Improved inference techniques must be implemented in fast algorithms, capable of producing n, T, and more, quickly, with modest computing resources.
  This rules out multi-physics 3D simulations, which would not be tractable in real time.

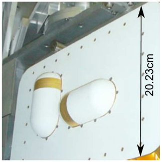
Solution

- Construct *solution libraries* consisting of measurements, with corresponding known plasma parameters.
- Such libraries could constructed experimentally if accurate inferences of experimental data were available.
- *Synthetic data* could also be made with simulations accounting for more physical processes and more realistic geometry than possible in theoretical models.
- Use these libraries to train and validate inference models.
- Two possibilities:
    - empirical analytic,
    - regression-based.

Example 1 - Swarm front plate as a PLP[3]

- When ion imagers are not in use, use the front plate as a PLP.
- Bias to $V = -3.5$ V, and measure the current.





$$A_{eff} = A_{geo}\left(1 + \delta_{model}\right)$$

$$\delta_{\mathrm{model}} = \frac{\alpha P \lambda_{\mathrm{D}}}{A_{\mathrm{geo}}}\left(1 - \frac{eV}{\frac{1}{2} m_{\mathrm{eff}} v_{\perp}^{2}} - \beta \frac{eV}{kT_{\mathrm{e}}} - \frac{\gamma}{eV}\frac{e^{2}}{4\pi\epsilon_{0}\lambda_{\mathrm{D}}}\right)$$

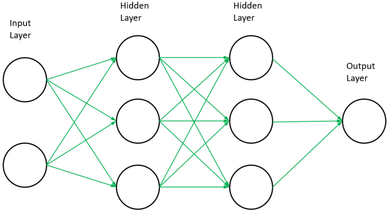
- $A_{eff} = A_{geo} \;\rightarrow\; nv_{ram}$ much too large.
- Ad hoc relative increases of A_{eff} by 8%, 12%, 17% $\rightarrow$ 11%, 9%, 14% errors
- Simulations and trained empirical model $\rightarrow\; < 2\%$ error.

[3] Resendiz Lira, et al., IEEE Trans. Plasma Sci., DOI: 10.1109/TPS.2019.2915216

Regression

Interpolate physical plasma parameters (n_e, T_e, , V_S, ...) in a multivariate space of physical measurements (I_i).

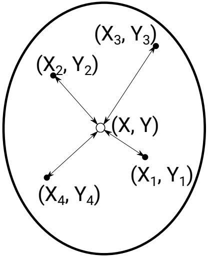
- Deep learning neural network. (Chalaturnyk, Marchand, Frontiers Phys. 2019)



- Radial basis functions.

$$\vec{Y} \simeq \sum_{i=1}^{N} a_i G\left(|\vec{X} - \vec{X}_i|\right)$$

$$\vec{Y}_j = \sum_{i=1}^{N} a_i G\left(|\vec{X}_j - \vec{X}_i|\right).$$



Two inference models

- Model 1: OML-like analytic, corrected with with multivariate regression.
- Model 2: Direct regression, no analytic bias.

In each case:

- Construct a data set with 4-tuples of currents and corresponding plasma parameters (n_e, T_e, V_f, ...)
- Use regression to construct an inference model for selected parameters.
- Train with a subset of our solution library.
- Validate with the remaining subset.

Example data set

For given bias voltages $V_b = 2,\ 3,\ 4,\ 5$ V:

I_1	I_2	I_3	I_4	n_e	T_e
-9.299e-07	-1.419e-06	-1.861e-06	-2.286e-06	2.514e+11	0.0563
-3.683e-09	-2.719e-07	-4.907e-07	-6.888e-07	8.146e+10	0.0541
-4.914e-07	-1.152e-06	-1.699e-06	-2.179e-06	2.872e+11	0.0554
-3.541e-08	-3.405e-07	-5.647e-07	-7.709e-07	8.271e+10	0.0929
-8.884e-08	-3.369e-07	-5.398e-07	-7.238e-07	7.564e+10	0.0518
-2.872e-07	-5.492e-07	-7.8107e-7	-1.009e-06	9.592e+10	0.0500
-1.52e-09	-3.223e-07	-6.253e-07	-8.944e-07	1.200e+11	0.0562
-9.599e-09	-5.003e-08	-8.305e-08	-1.139e-07	1.067e+10	0.0547

...

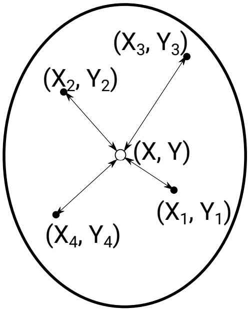
Radial basis function (RBF)

Given a data set, we need to choose

- the interpolating function G,
- the number and positions of pivots in $\vec{X}$ space,
- the coefficients a_i have to be determined.

For G, we can think of two possibilities:

- $G(0) \neq 0$ and $G(x)$ decreases monotonically for $x > 0$.
  $\rightarrow$ "*near neighbours*" interpolation.
- $G(0) = 0$ and $G(x)$ increases with $x > 0$.
  $\rightarrow$ "*all neighbours*" interpolation (as in kriging).



Training and validation

- Subdivide the full data set in two disjoint sets:
  - a training set, and
  - a validation set.
- Construct a model on the training set.
- Apply the model to the validation set
- In both cases, assess the model skill with a “loss” or “cost” function, which is
  - positive definite,
  - equal to 0 if model predictions are exact, and
  - increase as discrepancies between predictions and data increase.
- Avoid overfitting at training, which would lead to a loss of inference skill on the validation set.

Pivots and interpolation coefficients a_i

For a given number $\mathcal{N}$ of nodes in in a training set, two possible strategies are possible to select N pivots:

- Distribute pivots following the distribution of nodes in $\vec{X}$ space.
- Try all combinations N-choose-$\mathcal{N}$, and select the one which produces the best model over the entire set.

Coefficients a_i are determined by

- imposing collocation at pivots:

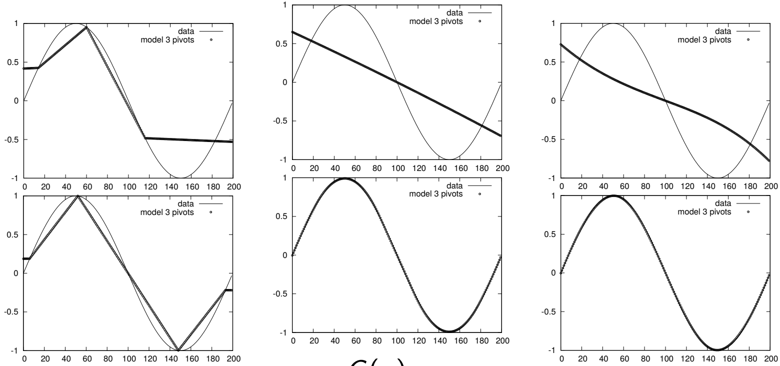
$$Y_i = \sum_{j=1}^{N} a_i G(|\vec{X}_i - \vec{X}_j|), \quad i = 1, N$$

When the best distribution of pivots is found (the cost function is smallest), collocation is relaxed to further improve the model skill.

Simple fit of sin(x)

$n_P = 3$

$n_P = 4$



$G(x) = |x|$

$G(x) = x^2 sign(1., x)$

$G(x) = |x^3|$

N.B.: The $G\left(|\vec{X} - \vec{X}_i|\right)$ functions must be linearly independent.

Combined analytic and regression

- There is an approximate analytic relation between currents I_i and plasma parameters.
- Use the analytic expression to infer approximate values of some of the parameters.
- Use regression to correct these estimates.

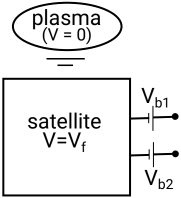
CAVEAT

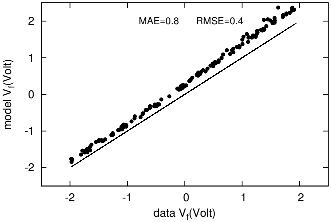
In order to do this, we need known, accurate values of the parameters in order to determine errors in analytic inferences, and correct them with regression.

Example 2 - fixed-bias spherical probes[4]

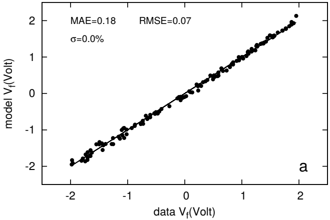
$$I = -ner^2\sqrt{\frac{8\pi kT_e}{m_e}}\left(1+\frac{e\left(V_f+V_b\right)}{kT_e}\right)$$

$$V_f = \frac{V_{b1}I_2 - V_{b2}I_1}{I_1 - I_2} - \frac{kT_e}{e}$$





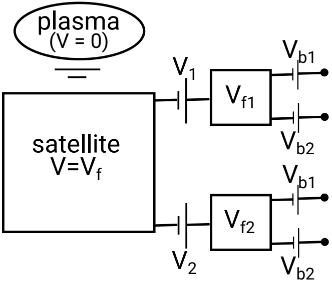
OML alone



Combined OML RBF

[4] Olowookere, Marchand, IEEE Trans. Plasma Sci. DOI: 10.1109/TPS.2020.3045366

Experimental validation



- Use two instruments biased to *different* and *variable* voltages V_1 and V_2.
- Verify whether

$$V_{f1} - V_1 = V_{f2} - V_2.$$

Example 3 - fixed-bias needle probes[5]

Assuming the empirical relation for the absolute value of the electron current collected by a finite length probe:

$$I \simeq n_e e A \sqrt{\frac{kT_e}{2\pi m_e}} \left(1 + \frac{e(V_f + V_b)}{kT_e}\right)^{\beta},$$

we can do the same as with fixed spherical probes and write

$$I^{1/\beta} \simeq \left(n_e e A \sqrt{\frac{k}{2\pi m_e}}\right)^{1/\beta} \frac{1}{(kT_e)^{1-1/(2\beta)}} \left(\frac{kT_e}{e} + V_f + V_b\right).$$

Given two probes biased to different voltages V_{b1} and V_{b2},

$$V_f + T_{eV} = \frac{V_{b1} I_2^{1/\beta} - V_{b2} I_1^{1/\beta}}{I_1^{1/\beta} - I_2^{1/\beta}}$$

[5] Guthrie, Marchand, Marholm, Meas. Sci. Technol., in press

Fixed-bias needle probes

- With 3 or more probes, β is determined from

$$\frac{V_{b1} I_2^{1/\beta} - V_{b2} I_1^{1/\beta}}{I_1^{1/\beta} - I_2^{1/\beta}} - \frac{V_{b3} I_2^{2/\beta} - V_{b2} I_3^{1/\beta}}{I_3^{1/\beta} - I_2^{1/\beta}} = 0.$$

- Given β, we can determine $V_f + T_{eV}$.
- Lastly, we can solve for $n_e / T_{eV}^{\beta-1/2}$ from

$$I \simeq \frac{n_e}{T_{eV}^{\beta-1/2}} A \sqrt{\frac{e^3}{2\pi m_e}} \left(T_{eV} + V_f + V_b\right)^{\beta}.$$

Construction of a model

Goal: Train more accurate model to infer plasma parameters from measured currents.

We need:

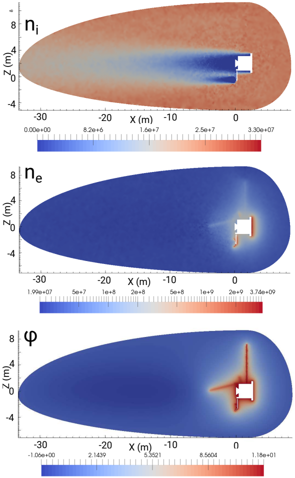
- A data set consisting of known plasma parameters (Level L2) to be inferred and corresponding low level L1B data (currents).
- A model which, given L1B data as input, will produce L2 data as output.
- A training and validation strategy.

How to construct a data base?

- Accurate experimental measurements would be ideal, but those are difficult to obtain.
- Synthetic data calculated with relevant physics, under representative conditions are good alternative.
- Apply 3D kinetic simulations to determine instrument (probe) responses in different assumed, and representative space environment conditions.
- Construct data sets, and train inference models using multivariate regression techniques.

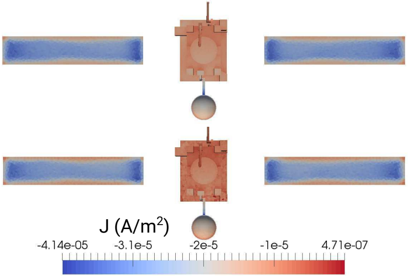
Solar Orbiter

(Grey, et al., IEEE Trans. Plasma Sci., 2017)



Early design of Solar Orbiter

Simulations made with PTetra



Effective collected current density without (above) and with (below) multiple reflections.

Method 1: OML-like (Guthrie, et al., in press)

Assuming OML-like characteristics,

$$I = -n_e e A \frac{2}{\sqrt{\pi}} \sqrt{\frac{kT_e}{2\pi m_e}} \left(1 + \frac{e(V_f + V_b)}{kT_e}\right)^{\beta},$$

$$I^{1/\beta} = \left[\left(n_e e A \frac{2}{\sqrt{\pi}} \sqrt{\frac{kT_e}{2\pi m_e}}\right)^{1/\beta} \frac{e}{kT_e}\right] \left(V_f + \frac{kT_e}{e} + V_b\right).$$

Solve for $V_f + kT_e/e$, given I_1, I_2, V_{b1}, V_{b2}:

$$V_f + \frac{kT_e}{e} = \frac{V_{b2} I_1^{1/\beta} - V_{b1} I_2^{1/\beta}}{I_2^{1/\beta} - I_1^{1/\beta}} = \frac{V_{b3} I_2^{1/\beta} - V_{b2} I_3^{1/\beta}}{I_3^{1/\beta} - I_2^{1/\beta}}.$$

V_f, T_e, and n_e

- With 3 probes at different voltages, β is the solution to:

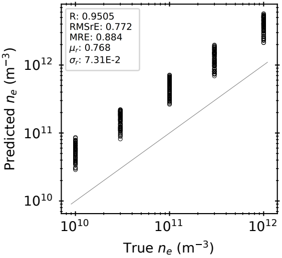
$$\frac{V_{b2} I_1^{1/\beta} - V_{b1} I_2^{1/\beta}}{I_2^{1/\beta} - I_1^{1/\beta}} - \frac{V_{b3} I_2^{1/\beta} - V_{b2} I_3^{1/\beta}}{I_3^{1/\beta} - I_2^{1/\beta}} = 0,$$

- Given β, estimate V_f with

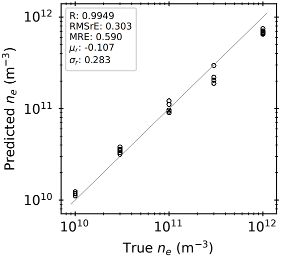
$$V_f \simeq \frac{V_{b2} I_1^{1/\beta} - V_{b1} I_2^{1/\beta}}{I_2^{1/\beta} - I_1^{1/\beta}} - T_e(eV).$$

- Correct with RBF. The correction is an estimate of $-T_e(eV)$.
- Use Eq. 1 to estimate n_e and correct with RBF.

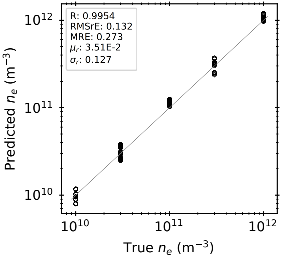
n_e: OML-like & RBF-corrected



Linear fit, assuming $\beta = 0.5$ (Hoang, et al, 1998)

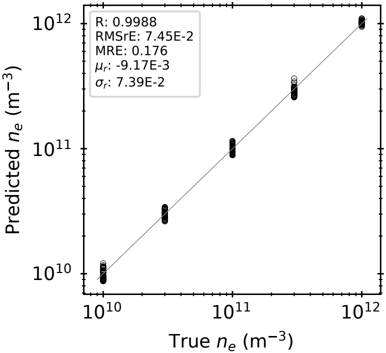


3-parameter nonlinear fit, exact T_e

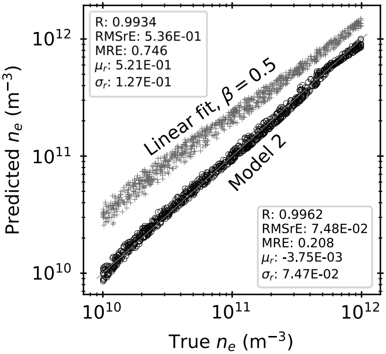


OML-like corrected with RBF T_e inferred.

Method 2: Direct RBF

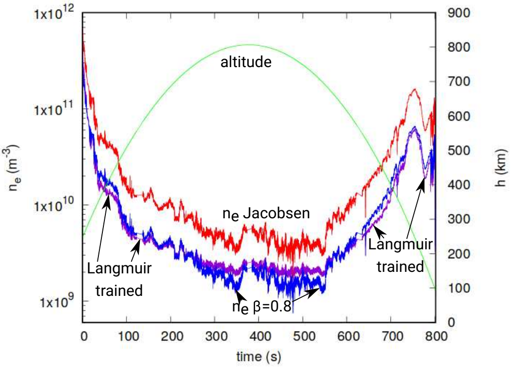


From PIC-simulated data set.
Direct RBF, 5 pivots



Langmuir model, 5 pivots
(Marholm 2019).

Application to Visions-2 data



Summary and conclusion

- After $\sim$ 100 years of research, measuring plasma parameters (density, temperature, potential, etc.), remains challenging.
- Plasma parameters are not measured directly. They are inferred from indirect L1B measurements.
- The inference of Langmuir probe measurements is essentially made with analytic expressions obtained from theories.
  - They often provide good estimates.
  - They are fast, and relatively simple.
- Approximations made in theories are generally not all satisfied.
- Direct computer simulations are not practical.
- A viable option is to:
  - Construct a solution library consisting of L1B data + plasma parameters.
  - Use these libraries to train and validate models to solve the inference problem.

Prospective: A change of paradigm?